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Combined Nonlinear Tokamak Plasma Current Profile Control System Design and Simulation with Input Constraints

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Abstract: The paper presents the design and simulation results of the combined multivariable control system with the plant input constraints for a plasma current profile in a tokamak. The plasma kinetic model was used in terms of the Grad-Shafranov plasma equilibrium equation and the poloidal magnetic flux diffusion equation. The model is square specifically contains 5 inputs and 5 outputs. Actuators are external current drive sources. Output is plasma current density measured at five points of the minor radius. A set of linear models for controller design was obtained by a state subspace identification approach. A set of H_∞ controllers was synthesized by a loop shaping methodology at various values of the plasma electron temperature on the magnetic axis. Controllers designed were applied by the numerical simulation to the original model to change the current profile from one state to another. In the presence of plant input constraints a special logic block was developed in addition to the linear robust controller and used when plant inputs saturated. The plasma current profile control system designed showed its operability in the tokamak plasma electron temperature range on the magnetic axis from 0.1 to 18 keV.

1. INTRODUCTION

Tokamaks (Artsimovich, 1972, Wesson, 1997) are leaders in the solution of a controlled thermonuclear fusion problem. In future thermonuclear tokamak-reactors the high temperature plasma has to be confined in magnetic field by magnetic control systems which have to keep plasma nearby the first wall with high accuracy and reliability (Ariola and Pironti, 2008, Mitrishkin *et al.*, 2011 and many others). As this takes place, plasma kinetic control systems have to generate and maintain optimal profiles namely of plasma current, safety factor, temperature, and density to provide stable and effective stationary regimes of tokamak-reactor operation. Such regimes are achieved by the additional heating of two types: neutral beam injection and electromagnetic waves (Wesson, 1997). In up-to-date tokamaks plasma magnetic control systems have been advanced in various aspects and give possibility to get plasma discharges in tokamaks, confine plasma for a long time, and keep progress in improving plasma parameters. But plasma kinetic control systems are only at the beginning of their development because the plasma kinetic control is much more complicated and on the other hand, the demand to develop plasma kinetic control systems appeared much later after the creation of the theoretical and experimental basis of the plasma magnetic control.

Open and closed-loop plasma kinetic control systems showed on a set of operating tokamaks such as JET (UK), DIII-D (US), ASDEX Upgrade (Germany), JT-60U (Japan), TCV (Switzerland), Tore Supra, (France) that desirable plasma profiles may be achieved under various conditions. But the experimental results exhibited difficulties to get desirable solutions. One of these challenges is to meet input saturations specifically to take into account input constraints in a control algorithm for a multivariable plant to achieve a control goal of the plasma profile control (Moreau *et al.*, 2009). For such plants each input influences on all outputs simultaneously. The present paper is devoted to that problem when the plasma current profile in a tokamak is under control by external current drive sources with output constraints which are the constraints of the plant inputs.

Section 2 provides short description of the plasma kinetic nonlinear model used for controller design and simulation. The problem of changing plasma current profiles by feedback control system is stated in Section 3. Section 4 deals with the state subspace identification methodology of the original model with distributed parameters. The design of a plasma current profile H_∞ controller together with a nonlinear logic block to meet plant input saturation conditions is represented in Section 5. Simulation results of the combined linear and nonlinear controller in the feedback of the kinetic plasma model are presented in Section 6. Conclusions are stated in Section 7.

2. PLASMA KINETIC MODEL

For tokamak plasma current profile control investigation the kinetic model is used which is given in (Khayrutdinov and Lukash, 1993, Mitrishkin *et al.,* 2008, 2011). In the model the Grad-Shafranov equation was applied to determine the plasma equilibrium in cylindrical coordinates (r, z, φ) centered on the axis of toroidal symmetry

$$r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\Psi}{\partial r}\right)+\frac{\partial^2\Psi}{\partial z^2}=-\frac{4\pi^2}{c}rj_t \qquad (1)$$

where Ψ is the poloidal flux function, c is the speed of light. In the right part j_t is the toroidal component of the plasma current density namely $j_t = r2\pi c\frac{\partial p}{\partial\Psi}+\frac{1}{rc}\frac{\partial\left(F^2\right)}{\partial\Psi}+j_{bs}+j_{CD}$ where p is the plasma pressure, F is the poloidal current, j_{bs} is the bootstrap current density and j_{CD} is the current density caused by the current drive of external sources. Functions $p = p(\Psi)$ and $F = F(\Psi)$ are found from the transport equations and the magnetic field diffusion equation specifically

$$\frac{d\Psi}{dt}\frac{\partial\Phi}{\partial\rho}-\frac{d\Phi}{dt}\frac{\partial\Psi}{\partial\rho}=\frac{4\pi}{\sigma}\left(J\frac{\partial F}{\partial\rho}-F\frac{\partial J}{\partial\rho}\right)+<\vec{j}\vec{B}>\frac{c}{\sigma}\frac{\partial V}{\partial\rho} \qquad (2)$$

where flux coordinate system (ρ, θ, φ) is exploited, Φ is the toroidal flux, $\rho=\sqrt{\Phi}$, θ and φ are poloidal and toroidal angles, $J(\rho)=\iint_{S_p} j_t drdz$ is the toroidal plasma current inside the S_p surface, σ is the plasma conductivity, $<\vec{j}\vec{B}>$ is the scalar product of the current density $\vec{j}$ and the magnetic field $\vec{B}$, $\left\langle\vec{j}\vec{B}\right\rangle=\left\langle\vec{j}\vec{B}\right\rangle_{bs}+\left\langle\vec{j}\vec{B}\right\rangle_{CD}$, $\left\langle\vec{j}\vec{B}\right\rangle_{bs}$ and $\left\langle\vec{j}\vec{B}\right\rangle_{CD}$ correspond to the bootstrap current and the current drive respectively. The plant input is the current density j_{CD} from the external current sources and the output is the plasma current density j_t. In the kinetic model which was used for plasma current profile control the plasma current density was measured at 5 points of the minor radius. The external current sources generate j_{CD} which are located and distributed around the same points.

3. CONTROL PROBLEM STATEMENT

The control of the kinetic parameters of plasma, including the current density profiles, is a topical issue for existing large tokamaks mentioned above and for being built tokamak-reactor ITER (France). The goal of plasma current profile control is a suppression of various types of MHD instabilities (Semenov *et al.*, 2006) leading to a disruption of a tokamak discharge and creating the optimal profile. For the plasma kinetic model (1), (2) a multivariable controller is supposed to be designed which should transfer the plasma current profile from one state to another in the range of plasma temperature on magnetic axis from 0.1 keV up to 18 keV.

To solve such a problem the first attempt was done in (Mitrishkin *et al.,* 2008) where the original plant model was identified on zero frequency and the decoupling controller was designed with the usage of the plant static model. But this approach made possible to control plasma current profile only in the range of the electron plasma temperature from 0.1 to 5 keV. This is because the static model did not give a chance to design a proper dynamic controller for the whole temperature range required. There is some progress in the kinetic controller design in (Mitrishkin *et al.,* 2011) where the controller was synthesised on the base of the dynamic identified model but without capability to deal with plant input saturations.

4. IDENTIFICATION OF ORIGINAL PLANT MODEL

The subspace identification problem is formulated for deterministic LTI systems given in the state space form in discrete time namely

$$x(k+1)=Ax(k)+Bu(k),\ \ y(k)=Cx(k)+Du(k) \qquad (3)$$

where $x(k)\in\mathbb{R}^n, u(k)\in\mathbb{R}^m, y(k)=\mathbb{R}^l$. Given a finite number of samples of the input signal $u(k)$ and the output signal $y(k)$ of the minimal realization system (3), the goal is to obtain the system matrices (A,B,C,D) and initial state vector up to a similarity transformation. The identification methodology is based on the fact that by storing the input and output data in structured block Hankel matrices it is possible to retrieve certain subspaces that are related to the system matrices. The system model is obtained in a non-iterative way via the solution of a number of linear-algebra problems. The key linear-algebra steps are an RQ factorization, an SVD, and the solution of a linear least-squares problem (Verhaegen and Verdult, 2007). The subspace methodology is realized by a family of N4SID numerical procedures (Ljung, 1999). In numerical simulation the inputs and outputs of the MIMO nonlinear plant model (1), (2) are sampled at discrete time points. Subsequently, the matrices of system (3) may be identified from these data.

Identification of the kinetic model (1), (2) is necessary to obtain a linear plant model suitable for designing a linear robust controller which will further operate on the original model. For identification purpose the input test signals were formed as rectangular waveforms with some displacement u_0 of each channel $u_k = u_m sign\left[\sin(\omega/k)\right]+u_0$ where $\omega=10\pi$, k=1..5, $u_m = 200$ A, $u_0 = 200$ A. The expansion of the rectangular waveform in a Fourier serious contains all harmonics and this makes possible to extract from the kinetic model the most representative data to build an adequate identified model. In this modeling test the actuator models were not taken into account. In doing so, scaling was used which allowed to equalize the plant outputs on the base of the expected amplitudes of each channel (Skogestad and Postlethwaite, 2005). Maximum input magnitudes are the same on all channels hence scaling is applied only to the outputs $y_k = \hat{y}_k / \hat{y}_{k\max}$ where output $\hat{y}_k$ is before scaling, $\hat{y}_{k\max}$ is the output maximum value of the channel k. The identified model order was chosen equal to 5 when the subspace identification procedure was applied. Such an order gave the minimal error between the outputs of the identified model and the nonlinear original plant in comparison with higher and lower orders tried.

As an example in Fig. 3 one can see the results of testing of both models by rectangular waveform signals of different frequency of each channel at the plasma temperature 10 keV. The output signal curves of original and identified models were superimposed. The differences between outputs of two plant models were numerically estimated by the relative integral deviation criterion $Q = \sqrt{\int_{t_0}^{t_1} (y_1(t) - y_2(t))^2 dt / \int_{t_0}^{t_1} y_2^2 dt}$ where y_1 and y_2 are outputs of identified and original plant models respectively. The estimates of the model comparison in accordance with the criterion Q are listed in Table 1 at various values of the plasma temperature T on the magnetic axis specifically 0.1; 3; 5; 10 keV. The worst result of the identification was obtained at the lowest temperature of 0.1 keV in the range from 33% up to 40% for various channels. For other temperature values the identification accuracy is noticeably better and is located in the range from 7% up to 13%. But this identification accuracy was sufficient to build robust linear controller which dealt with the problem of the plasma current profile change from one state to another.

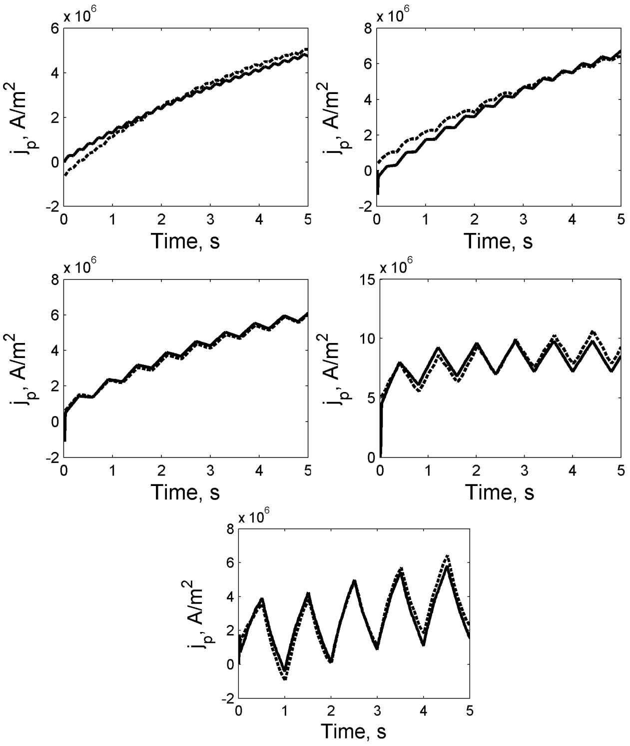


Fig. 3. Output signals of identification procedure from original plant model (solid lines) and identified model (dotted lines) of control channels 1-5 at the plasma temperature 10 keV

Table 1. Deviation estimates of identified model outputs

Estimate, %	Temperature, keV			
Channel No	**0.1**	**3**	**5**	**10**
1	40.2	8.93	5.98	8.64
2	36.2	12.3	6.49	9.31
3	33.3	13	6.74	7.63
4	33.4	12.2	6.9	7.43
5	35.5	12.2	8.29	8.18

5. MULTIVARIABLE CONTROLLER DESIGN

The multivariable feedback controller design process consists of two stages. At the first stage a linear controller is designed to cope with the original plant model having 5 inputs/5 outputs without input constraints. At this stage the loop shaping methodology of (McFarlane and Glover, 1992) was used to design robust controller with an acceptable robust stability margin to overcome model uncertainty obtained after the identification procedure. The second stage was devoted to the design of the nonlinear part of the controller to cope with plant input saturations. In this case, special control logic was developed to provide restrictions of control actions when a transition of plasma current profiles is applied.

The loop-shaping design procedure is a two phase design process. First, the open-loop plant is augmented by pre- and post-compensators W_1 and W_2 respectively to give a desired shape to the singular values of the open-loop frequency response. In our case this plant model is the identified model obtained as a result of the identification procedure. Then the resulting shaped plant is robustly stabilized with respect to coprime factor uncertainty using the H_∞ optimization approach. We will consider the stabilization of a plant G which has a normalized left coprime factorization $G = M^{-1}N$ where N and M are stable coprime matrix transfer functions. The objective of the robust stabilization is to stabilize by the H_∞ feedback controller K_∞ not only the nominal model G but a family of perturbed plants defined by

$$G_p = W_2 G W_1 = \{(M + \Delta_M)^{-1}(N + \Delta_N) : \| [\Delta_M \quad \Delta_N] \|_\infty < \varepsilon\}$$

where ε is a robust stability margin that shows the maximum possible H_∞-norm of model uncertainty $[\Delta_M \quad \Delta_N]$ at which the closed-loop system keeps stability. The model uncertainty represents the fact of not full knowledge of plasma nonlinearities, parameters, and diagnostics. The final controller K has the form: $K = W_1 K_\infty W_2$.

The original plant model was given in discrete time so the identified model was converted into continuous time for application of robust controller design techniques. In controller design procedure the pre-compensator W_1 was used as a diagonal transfer matrix with integral units $W_1(s) = diag(1/s)_1^5$ to give the designed system the astatic feature in each channel. Post-compensator W_2 was chosen as a diagonal matrix with coefficients which were adjusted to achieve equal time responses of each channel. Acceptable stability margins ε were received for controllers designed for various plasma temperatures as follows: 0.51 at 0.1 keV, 0.39 at 3 keV, and 0.38 at 18 keV. So designed controllers are able to cover the whole plasma temperature range when plasma current profile transitions in the presence of plant uncertainties $[\Delta_M \quad \Delta_N]$.

The block diagram of the closed-loop control system is presented in Fig. 4 where the vector output signal y is plasma current density J [A/m^2] at 5 points of minor radius; the plant input vector v consists of 5 input actions from current drive sources.

When there is a saturation block at the plant input then an additional logic was developed to prevent input signals to exceed the limits. So the logic block in a nonlinear part of the controller prevents the accumulation of signals in the controller. The controller output which comes to the saturation block must not exceed the established limit of 15 kA. The prevention of the accumulation means that if the controller output u exceeds the limit and the sign of the error e is the same as the sign of the signal u, the error e will be blocked by the logic unit that is becomes equal to zero (Fig. 5). The logic block accepts the signal u from the linear controller output and the error signal e to create a new signal e_m for the linear controller input.

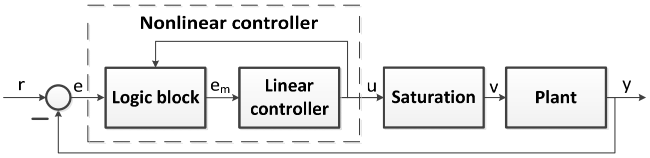


Fig. 4. Block diagram of control system with input constraints (saturations) and nonlinear controller

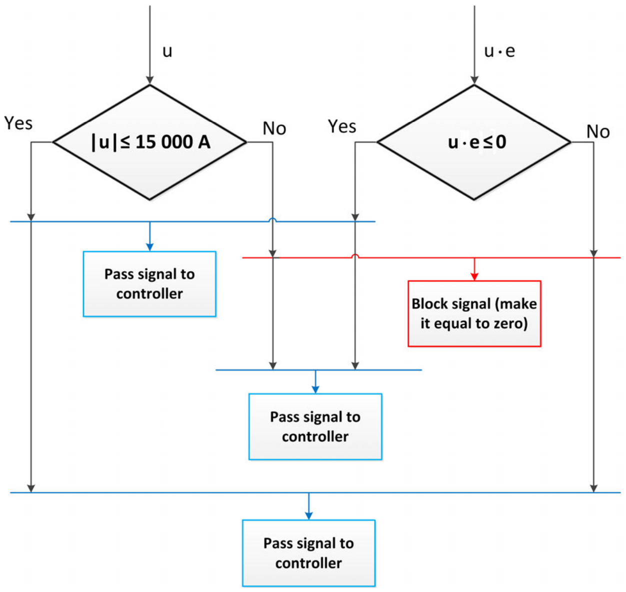
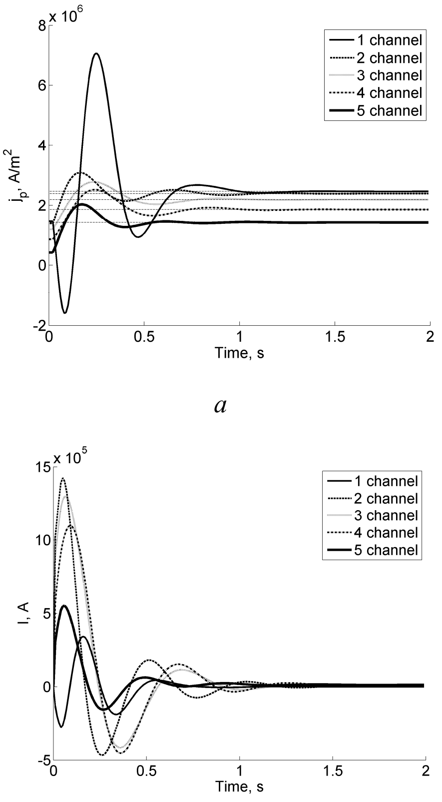


Fig. 5. Operating principle of the logics block

6. CONTROL SYSTEM SIMULATION

First, the linear controllers designed were simulated in the feedback of the original plant model for the relevant plasma temperatures without input constraints and the nonlinear logic block. As an example in Fig. 6 the plant outputs and inputs are represented at T=18 keV because this temperature corresponds to the burning regime of a thermonuclear reactor. The output signals in Fig. 6(a) converge to their steady-state values during approximately 1 s when the transition from the initial plasma current profile to its predetermined value (reference). The inputs during the transition converge to zeros (Fig. 6(b)).

The initial and final plasma current profiles are presented in Fig. 7. In the case of the plant kinetic model (1), (2) the initial profile is a smooth line because it is formed by an initial distribution of a bootstrap plasma current caused by a density gradient (Wesson, 1997). Then the control system transfers the outputs to their references and the steady-state errors are equal to zeros at all channels because the system has integrating units in the feedback which appeared from the pre-compensator in the controller design procedure.



a

b

Fig. 6. Plant output (a) and input (b) signals at the temperature 18 keV

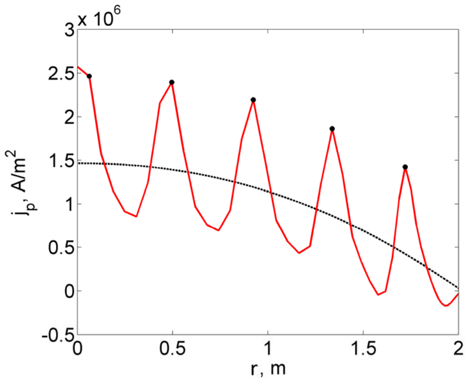


Fig. 7. Profiles of current density: the initial (black dotted line) and final (red solid line) at the temperature of 18 keV, asterisks mark the desired (reference) profile

But the plasma current density between measured outputs on the minor radius has obvious dips and the plasma current profile has a wave form. The reason of this phenomenon is in the model assumption namely the full plasma current is fixed.

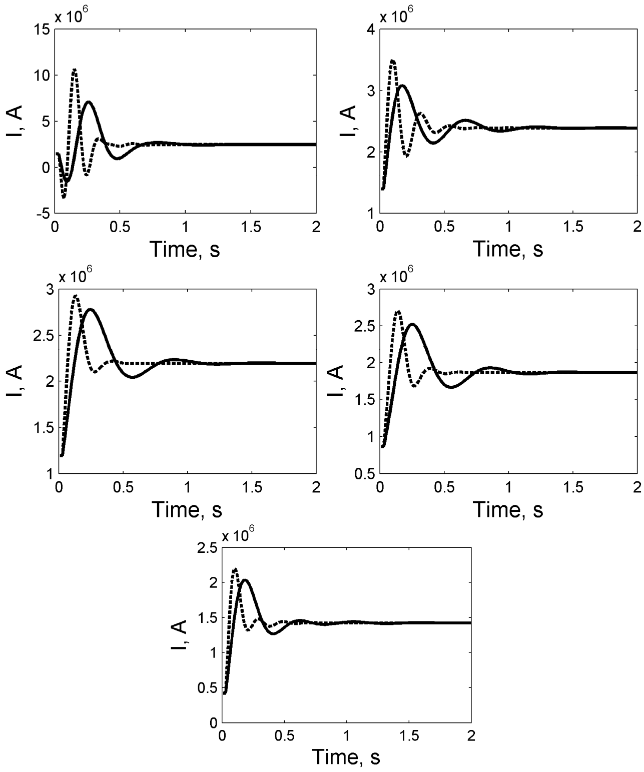


Fig. 8. Output signals at 18 keV (solid) and 10 keV (dotted)

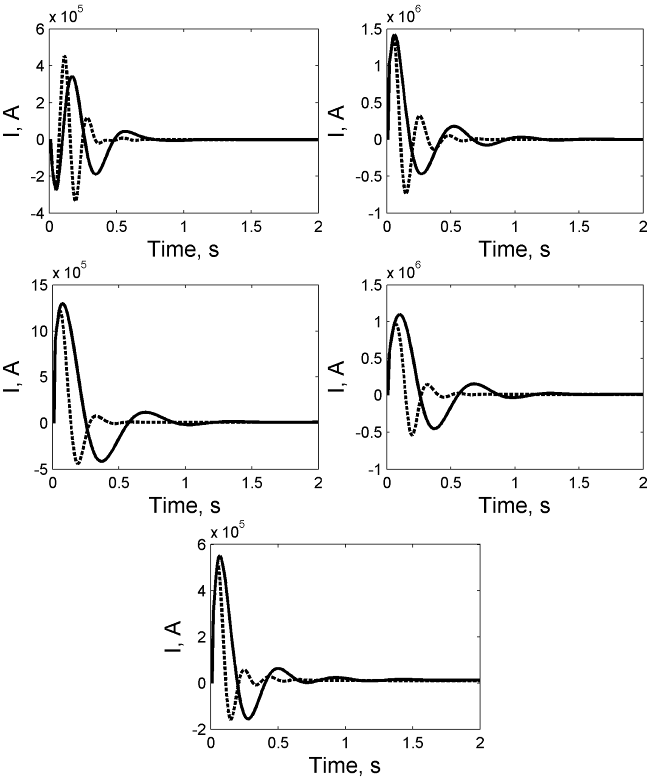


Fig. 9. Input signals at 18 keV (solid) and 10 keV (dotted)

So when the system acts on the plasma current density by means of current drive actions and the plasma current is fixed the current density between measuring points has to decrease. The other reason of the phenomenon is a distribution nature of outputs of current drive sources and superposition of these distributions during plasma current control. In spite of the phenomenon the control system demonstrated the acceptable operability with the distributed parameter plant and achieved the objective of the work given.

During the system simulation it was discovered that the controller designed for *T*=10 keV is capable of operation in the temperature range from 10 to 18 keV. The comparison of the controller operation at 10 keV and 18 keV is shown in Fig. 8 (outputs) and in Fig. 9 (inputs). The transient response time in the case of 18 keV is larger because the plant at higher temperature becomes more inertial.

For the investigation of the influence of constrained input values on the duration of the transient responses when the references changed the system was simulated in the presence of the saturation block (Fig. 4) for two cases: with and without logic block. The results are listed in Table 2.

Table 2. Response times (s) at various input constraints (I_c) with (L) and without (WL) logic block

I_c, kA	15		12		10	
T, keV	L	WL	L	WL	L	WL
3	1.5	3	2.5	5	5	19
10	6	24	10	34	30	40
18	13	75	22	90	60	120

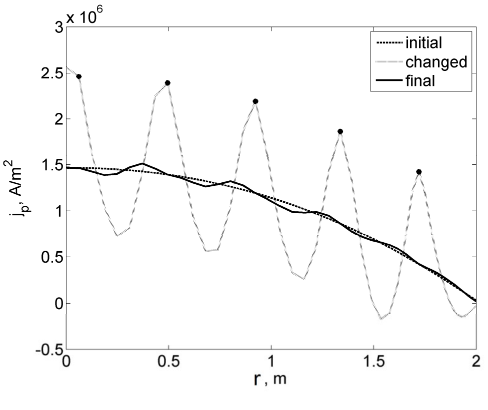


Fig. 10. Profiles of the plasma current density at 18 keV using the logic block, asterisks mark the desired profile of the changed state.

Various values of actuator maximum currents were used specifically 10, 12, and 15 kA accompanied by the set of plasma temperatures namely 3, 10, 18 keV. One can see from Table 2 that the effectiveness of the logic block increases when the temperature increases. On the other hand, the decrease of the input constraint from 15 kA to 10 kA increases the response time by an undesirable manner. So the best version is the presence of the logic block at the constraint of 15 kA.

The final demonstration of the system operationability is the operation at the plasma temperature of 18 keV, the input restriction of 15 kA, in the presence of logic block and the reference change from initial state $[0.1463\ \ 0.1391\ \ 0.1192\ \ 0.0864\ \ 0.0423\ \ 0.0003]\times 10^7$ A/m^2 to the state $[0.2463\ \ 0.2391\ \ 0.2192\ \ 0.1864\ \ 0.1423\ \ 0.0997]\times 10^7$ A/m^2 and back (Fig. 10, 11).

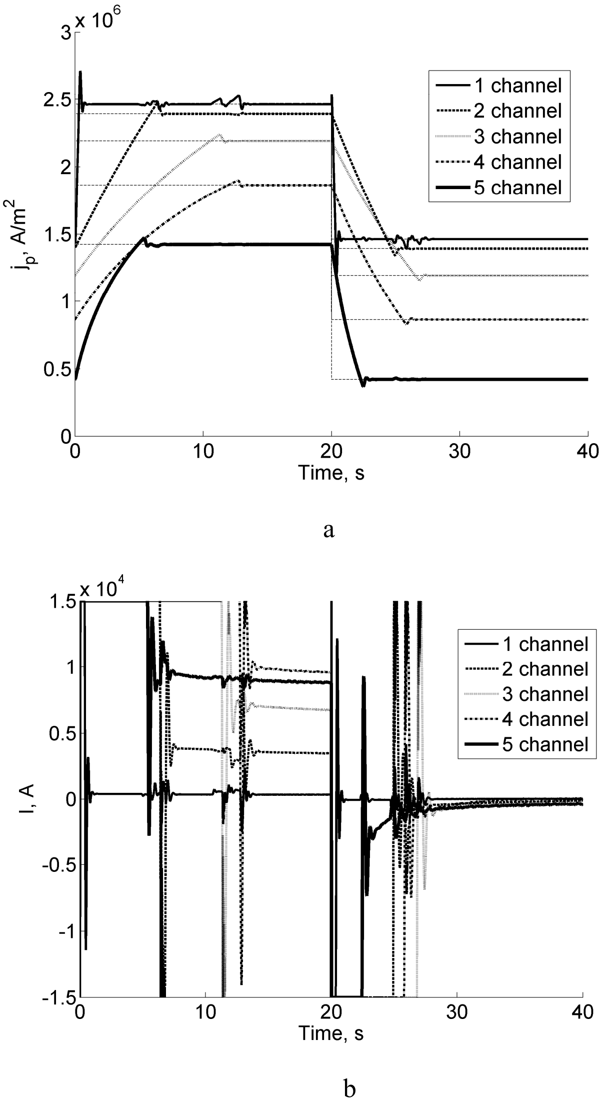


Fig. 11. Plant output (a) and input (b) signals at the temperature of 18 keV using the logic block when profile reference is changed from initial state and back

7. CONCLUSIONS

For the plasma kinetic model on the base of the poloidal flux diffusion equation the control system was designed and simulated to transfer the plasma current profile from one state to another. The final controller designed contains the combination of the linear robust part and nonlinear logic block to cope with plant constraints. The simulation results demonstrated the controller operability in the presence of input constraints and full plasma current restriction assumed in the plant model with distributed parameters. The range of plasma electron temperature on the magnetic axis for the tested controller set was from 0.1 keV up to 18 keV. The maximum temperature value 18 keV corresponds to a tokamak reactor regime when a thermonuclear reaction takes place.

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